2.3.1 Cramer's rule and matrix model

Cramer's rule is used for the solution of linear equations by determinants. Let the system of linear equations in three unknown I_1,I_2 and I_3 are:

$$R_{11}I_1 + R_{12}I_2 + R_{13}I_3 = V_1$$
$$R_{21}I_1 + R_{22}I_2 + R_{23}I_3 = V_2$$
$$R_{31}I_1 + R_{32}I_2 + R_{33}I_3 = V_3$$

May be written in matrix from as:

$$\begin{pmatrix} R_{11} & R_{12} & R_{13} \\ R_{21} & R_{22} & R_{23} \\ R_{31} & R_{32} & R_{33} \end{pmatrix} \begin{pmatrix} I_1 \\ I_2 \\ I_3 \end{pmatrix} = \begin{pmatrix} V_1 \\ V_2 \\ V_3 \end{pmatrix}$$

Where R_{ii} = the total resistance of the i^{th} loop with +ve sign.

R_{ij} = common resistance between i^{th} loop and j^{th} loop with –ve sign.

Then by Cramer's rule, the solution of these simultaneous equations is given by:

$$I_1 = \frac{\Delta_1}{\Delta}, I_2 = \frac{\Delta_2}{\Delta} \text{ and } I_3 = \frac{\Delta_3}{\Delta}$$

Where,

$$\Delta = \left| \begin{pmatrix} R_{11} & R_{12} & R_{13} \\ R_{21} & R_{22} & R_{23} \\ R_{31} & R_{32} & R_{33} \end{pmatrix} \right|$$

$$\Delta_1 = \left| \begin{pmatrix} V_1 & R_{12} & R_{13} \\ V_2 & R_{22} & R_{23} \\ V_3 & R_{32} & R_{33} \end{pmatrix} \right|$$

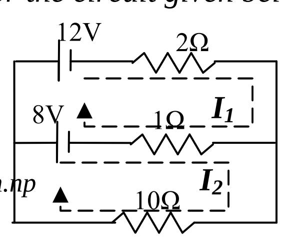
$$\Delta_2 = \left| \begin{pmatrix} R_{11} & V_1 & R_{13} \\ R_{21} & V_2 & R_{23} \\ R_{31} & V_3 & R_{33} \end{pmatrix} \right| \text{ and}$$

$$\Delta_3 = \left| \begin{pmatrix} R_{11} & R_{12} & V_1 \\ R_{21} & R_{22} & V_2 \\ R_{31} & R_{32} & V_3 \end{pmatrix} \right|$$

Cramer's rule can be used for solving simultaneous equations if the numbers of unknown are more than two. This gives quick results.

Example: 2

Generate the matrix model for the circuit given below for the branch currents.



Ans:

$$\begin{pmatrix} R_{11} & R_{12} \\ R_{21} & R_{22} \end{pmatrix}\begin{pmatrix} I_1 \\ I_2 \end{pmatrix} = \begin{pmatrix} V_1 \\ V_2 \end{pmatrix}$$

Or

$$\begin{pmatrix} 2+1 & -1 \\ -1 & 10+1 \end{pmatrix}\begin{pmatrix} I_1 \\ I_2 \end{pmatrix} = \begin{pmatrix} -12+8 \\ -8 \end{pmatrix}$$

Or

$$\begin{pmatrix} 3 & -1 \\ -1 & 11 \end{pmatrix}\begin{pmatrix} I_1 \\ I_2 \end{pmatrix} = \begin{pmatrix} -4 \\ -8 \end{pmatrix}$$

Example: 3

Solve the following three simultaneous equations using Cramer's rule.

$$I_1+3I_2+4I_3=14$$
$$I_1+2I_2+I_3=7$$
$$2I_1+I_2+2I_3=2$$

Ans:

As explained earlier, the matrix model for the given circuit can be written in the form:

$$\begin{pmatrix} 1 & 3 & 4 \\ 1 & 2 & 1 \\ 2 & 1 & 2 \end{pmatrix}\begin{pmatrix} I_1 \\ I_2 \\ I_3 \end{pmatrix} = \begin{pmatrix} 14 \\ 7 \\ 2 \end{pmatrix}$$

$$\Delta = \left|\begin{pmatrix} 1 & 3 & 4 \\ 1 & 2 & 1 \\ 2 & 1 & 2 \end{pmatrix}\right| = 1(4-1)-1(6-4)+3(3-8) = -9$$

$$\Delta_1 = \left|\begin{pmatrix} 14 & 3 & 4 \\ 7 & 2 & 1 \\ 2 & 1 & 2 \end{pmatrix}\right| = 14(4-1)-7(6-4)+2(3-8) = 18$$

$$\Delta_2 = \left|\begin{pmatrix} 1 & 14 & 4 \\ 1 & 7 & 1 \\ 2 & 2 & 2 \end{pmatrix}\right| = 1(14-2)-1(28-8)+2(14-28) = -36$$

$$\Delta_3 = \left|\begin{pmatrix} 1 & 3 & 14 \\ 1 & 2 & 7 \\ 2 & 1 & 2 \end{pmatrix}\right| = 1(4-7)-1(6-14)+2(21-28) = -9$$

$\therefore$ According to Cramer's rule,

$$I_1 = \frac{\Delta_1}{\Delta} = \frac{18}{-9} = -2A; I_2 = \frac{\Delta_2}{\Delta} = \frac{-36}{-9} = 4A; I_3 = \frac{\Delta_3}{\Delta} = \frac{-9}{-9} = 1A$$