2.9 Nodal Analysis

In this method, one node of the circuit is taken as lowest potential reference at zero volts (called circuit ground); while the voltage of other nodes are assumed to be $V_1, V_2, V_3 \ldots$ with respect to the reference node.

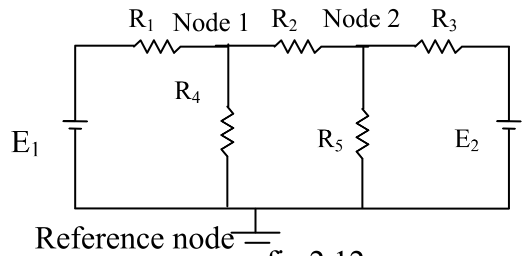


fig.2.12

The current equations are written by using KCL. Thus, if there is n number of nodes in the network, then current equations for (n-1) nodes are written and then solved.

Consider a network as shown in fig.2.12. Node 3 has been taken as reference node. Let V_1 and V_2 be the voltages of the nodes 1 and 2 respectively with respect to reference node. Then applying KCL at node 1, we get:

$$\frac{V_1 - E_1}{R_1} + \frac{V_1 - V_2}{R_2} + \frac{V_1}{R_4} = 0$$

Or

$$\left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_4}\right)V_1 - \frac{1}{R_2}V_2 = \frac{E_1}{R_1} \qquad (2.11)$$

Similarly applying KCL at node 2, we get:

$$\frac{V_2 - V_1}{R_2} + \frac{V_2}{R_5} + \frac{V_2 - E_2}{R_3} = 0$$

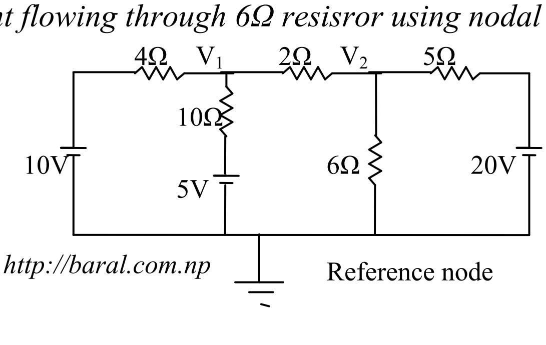
Or

$$-\frac{1}{R_2}V_1 + \left(\frac{1}{R_2} + \frac{1}{R_3} + \frac{1}{R_5}\right)V_2 = \frac{E_2}{R_3} \qquad (2.12)$$

Then the equations 2.11 and 2.12 can now be solved to get the value of V_1 and V_2. Then, the value of branch current can be calculated easily.

Example: 5

Find the current flowing through 6Ω resisror using nodal analysis.



Ans:

Applying KCL at node 1:

Or $$\left(\frac{1}{10}+\frac{1}{2}+\frac{1}{4}\right)V_1-\frac{1}{2}V_2=\frac{5}{10}+\frac{10}{4}$$

$$0.85V_1-0.5V_2=3 \qquad (1)$$

$$-\frac{1}{2}V_1+\left(\frac{1}{5}+\frac{1}{2}+\frac{1}{6}\right)V_2=\frac{20}{5}$$

$$-0.5V_1+0.867V_2=4 \qquad (2)$$

By solving equations (1) and (2), node voltages can be calculated as follows:

$$V_1=$$

$$V_2=$$

Case: 1

If the circuit contains independent current source only:

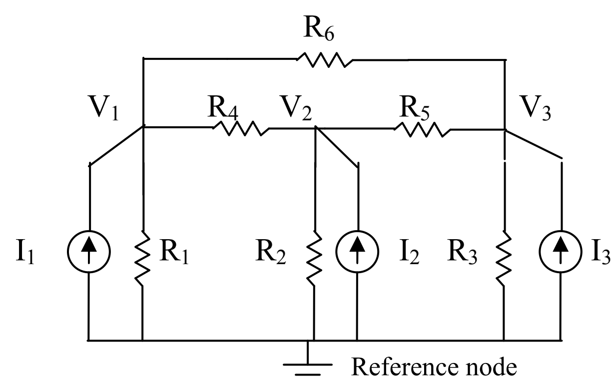


Fig.2.13

Using KCL at node (1) we get:

$$\frac{V_1-V_2}{R_4}+\frac{V_1-V_3}{R_6}+\frac{V_1}{R_1}=I_1$$

Or

$$\left(\frac{1}{R_4}+\frac{1}{R_6}+\frac{1}{R_1}\right)V_1-\frac{1}{R_4}V_2-\frac{1}{R_6}V_3=I_1 \qquad (2.13)$$

Using KCL at node (2) we get:

$$\frac{V_2-V_1}{R_4}+\frac{V_2-V_3}{R_5}+\frac{V_2}{R_2}=I_2$$

Or

$$-\frac{1}{R_4}V_1+\left(\frac{1}{R_2}+\frac{1}{R_4}+\frac{1}{R_5}\right)V_2-\frac{1}{R_5}V_3=I_2 \qquad (2.14)$$

Again, using KCL at node (3) we get:

$$\frac{V_3-V_2}{R_5}+\frac{V_3-V_1}{R_6}+\frac{V_3}{R_3}=I_3$$

Or

$$-\frac{1}{R_6}V_1-\frac{1}{R_5}V_2+\left(\frac{1}{R_6}+\frac{1}{R_3}+\frac{1}{R_5}\right)V_3=I_3 \qquad (2.15)$$

$\therefore$ from above theree equitations:

$$\begin{pmatrix}G_{11} & G_{12} & G_{13}\\ G_{21} & G_{22} & G_{23}\\ G_{31} & G_{32} & G_{33}\end{pmatrix}\begin{pmatrix}V_1\\ V_2\\ V_3\end{pmatrix}=\begin{pmatrix}I_1\\ I_2\\ I_3\end{pmatrix} \text{ Or } \quad [G][V]=[I]$$

Where, G_{ii} is the sum of total conductance connected to the node I with positive sign.
G_{ij} is the conductance connected between node i and node j with negative sign.
I_i is the Magnitude of ideal current source (incoming current is taken as positive).

Case: 2

If the independent ideal voltage source is connected between the reference node and any of non-reference nodes:

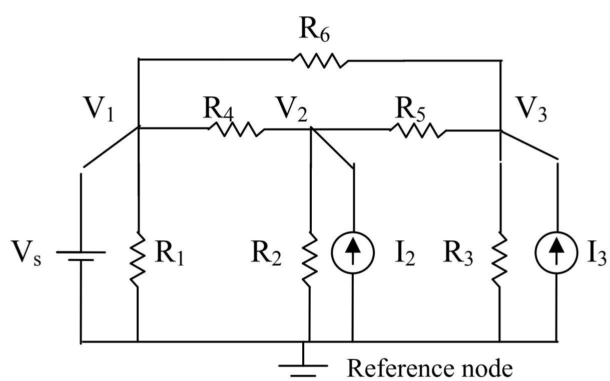


Fig2.14

If an independent ideal voltage source is present between the references node and any of the non-reference nodes say node 1, then the voltage level of node 1 is equal to the voltage of the ideal voltage source.

$$\therefore \quad V_1 = V_s$$

And equations for other nodes can be written simply as case 1.

Case: 3
If the independent ideal voltage source is connected between two non-reference nodes:

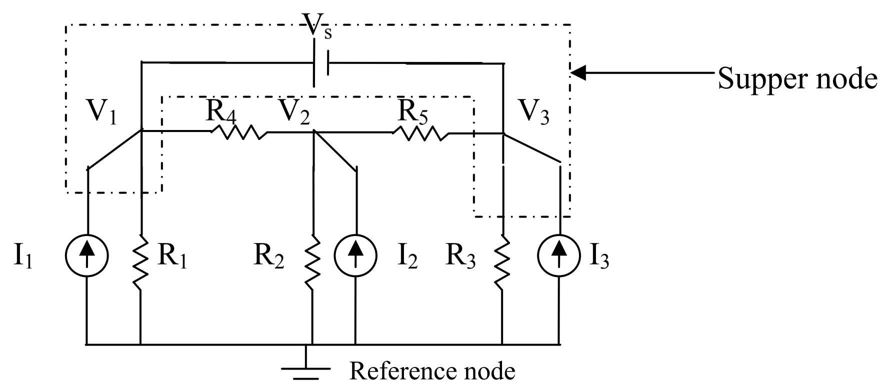


Fig2.15

Consider a circuit as shown in fig.2.15, in which an ideal voltage source Vs is connected between node1 and node 2. For node 2 we can simply write the KCL equation.
Applying KCL for node (2):

$$\frac{V_2 - V_1}{R_4} + \frac{V_2 - V_3}{R_5} + \frac{V_2}{R_2} = I_2 \qquad (1)$$

But, node 1 and node 3 are connected by an ideal voltage source, therefore we can make a super node by adding the node 1 and node 3 and neglecting the voltage source 'temporarily'.
Applying KCL for Super node:

$$\frac{V_1 - V_2}{R_4} + \frac{V_1}{R_1} - I_1 + \frac{V_3 - V_2}{R_5} + \frac{V_3}{R_3} - I_3 = 0 \qquad (2)$$

And also,

$$V_1 - V_s - V_3 = 0 \qquad (3)$$

By solving the equations we can find the node voltages and hence the branch currents flowing through the circuit.

