Resonance in Series R-L-C Circuit

Series or Voltage Resonance:

Consider an ac circuit containing resistance of R ohms, inductance of L henrys and capacitance of C farads connected in series, as shown in fig.3.21 (a).

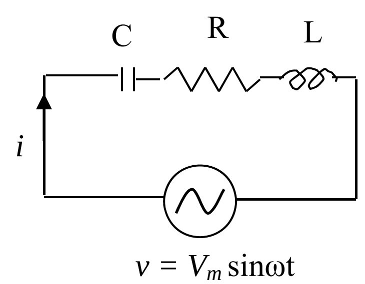
Impedance of the circuit

$$Z = \sqrt{R^2 + \left(X_L - X_C\right)^2} = \sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}$$

If for some frequency f applied voltage, $X_L = X_C$ in magnitude then

- Net reactance of the circuit is zero i.e. X = 0
- Impedance of the circuit, Z = R
- The current flowing through the circuit is maximum and in phase with the applied voltage. The magnitude of current will be equal to V/R.
- The voltage drop across the inductance is equal to the voltage drop across the capacitance i.e. $V_L = V_C$.
- Supply voltage is equal to the voltage drop across the resistance i.e. $V = V_R$.
- Power expended = V I watts and
- Power factor is unity.

When this condition exists, the circuit is said to be in series resonance and the frequency at which it occurs is known as resonance frequency.



(a) Circuit diagram

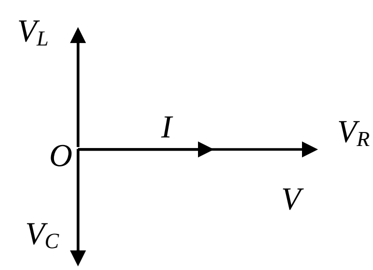


Fig.3.12 (b) Phasor diagram

Fig.3.21

If the resonance frequency is denoted by f_o, then

$$X_L = \omega L = 2\pi f_o L$$

and $$X_C = \frac{1}{\omega C} = \frac{1}{2\pi f_o C}$$

Since for resonance $X_L = X_C$

$$\therefore \quad 2\pi f_o L = \frac{1}{2\pi f_o C}$$

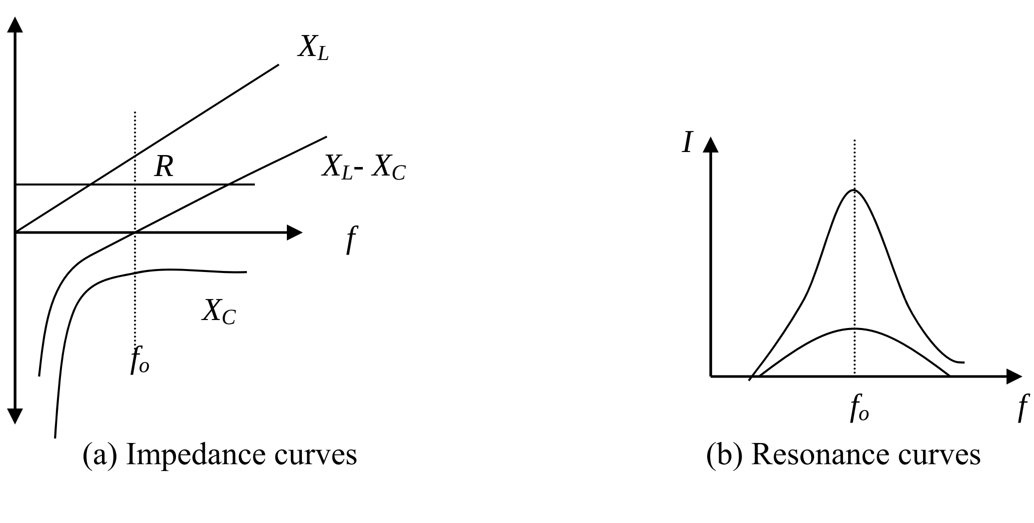
or $$f_o = \frac{1}{2\pi\sqrt{LC}}$$

or $$\omega_o = \sqrt{\frac{1}{LC}}$$

From the above expression it is clear that the value of resonance frequency depends on the parameters of two energy-storing elements.

For any frequency lower than resonant frequency f_o, inductive reactance is lesser than the capacitive reactance and so the circuit behaves like capacitive circuit. Similarly for any frequency grater than resonant frequency f_o, reactance is grater than the capacitive reactance and so the circuit behaves like inductive circuit. When the applied frequency is equal to the resonant frequency, the inductive reactance is equal to the capacitive reactance; the voltage drop across the inductance is equal to the voltage drop across the capacitance in magnitude, but opposite in the phase and, therefore, the circuit current I is in phase with the applied voltage i.e. the circuit behaves like a resistive circuit.

When the circuit is in resonance, the circuit current is too large and will produces large voltage drop across the inductance and capacitance. If resistance R would have not been present in the circuit, such circuit, would act like short-circuit to currents of frequency to which it resonates.



(a) Impedance curves (b) Resonance curves

Fig.3.22

Current varies inversely with the variation in impedance and, therefore, it is maximum at resonance frequency when the impedance is minimum and decrease with the variation in frequency on both side of the resonance frequency as shown in fig.3.22(b).

The curve drawn between the circuit current and the frequency of the applied voltage is called the resonance curve and its shape depends upon the value of circuit resonance R as shown in fig. For smaller value of resistance the curve is sharply peaked, but for larger value of R, it is flat.

Quality factor is the voltage magnification factor at resonance. It is also defined as the ratio of resonance frequency to band width. It is a measure of selectivity or sharpness of tuning of series RLC circuit.

$$Q \text{ factor } = \frac{X_L}{R} = \frac{2\pi f_o L}{R} = \frac{2\pi f_o}{R}\frac{1}{2\pi\sqrt{LC}}$$

$$= \frac{1}{R}\sqrt{\frac{L}{C}}$$

Thus the quality factor can be increased by decreasing R or increasing the L/C ratio.

Selectivity and Bandwidth:

We have seen that for low resistance circuit the resonance curve is shapely peaked and such circuit is said to be sharply-resonant or *high-selective*. On the other hand, high resonance circuit has flat resonance curve and said to have poor selectivity. Selectivity of different resonant circuit are compared in terms of their bandwidths, which are given by the bands of frequencies which lie between two points on either side of the resonant frequency where the current is $\frac{1}{\sqrt{2}}$ times of the maximum current I_{max}.

At resonance frequency power dissipated on the circuit is $P = I_{\max}^2 R$

And at frequencies f_1 and f_2 where current is $\frac{1}{\sqrt{2}} I_{\max}$, power $P' = \frac{I_{\max}^2 R}{2}$ i.e. half power at resonant frequency. These frequencies f_1 and f_2 at which power dissipation on the circuit is half of the power at resonant frequency are known as half power frequencies.

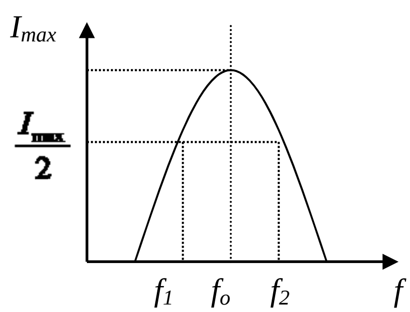


Fig.3.23 Band width of series resonance circuit

For RLC series circuit

$$I = \frac{V}{\sqrt{R^2 + (X_L - X_C)^2}} = \frac{\frac{V}{R}}{\sqrt{1 + \left(\frac{X_L - X_C}{R}\right)^2}}$$

$$I = \frac{I_{max}}{\sqrt{1 + \left(\frac{X_L - X_C}{R}\right)^2}} \qquad (1)$$

But at half power frequencies $I_1 = I_2 = \frac{I_{max}}{\sqrt{2}}$ (2)

Therefore from eq.(1) and (2)

$$\frac{I_{max}}{\sqrt{2}} = \frac{I_{max}}{\sqrt{1 + \left(\frac{X_L - X_C}{R}\right)^2}}$$

$$\sqrt{2} = \sqrt{1 + \left(\frac{X_L - X_C}{R}\right)^2}$$

$$2 - 1 = \left(\frac{X_L - X_C}{R}\right)^2$$

$$\therefore R = X_L - X_C \qquad (3)$$

But at resonance $X_L - X_C = 0$

When frequency is increased from f_o to f_2; X_L must increased by 0.5R and X_C must correspondingly decreased by 0.5R to satisfy the eq.(1).

Therefore

$$X_{L2} - X_{Lo} = 0.5R$$

$$or\ 2\pi f_2 L - 2\pi f_o L = 0.5R$$

$$or\quad f_2 - f_0 = \frac{R}{4\pi L}$$

Similarly,

$$f_o - f_1 = \frac{R}{4\pi L}$$

$$\therefore \text{Bandwidth } (\Delta f) = \frac{R}{4\pi L} + \frac{R}{4\pi L} = \frac{R}{2\pi L}$$